Ideas for Math Extended Essays

The entries in this file are just titles or brief descriptions for a pile of projects. This is meant as a browsing list. Seeing a title here might give you an idea for a project. For that reason I have been careful not to classify the titles by subject area.

1. Does the great pyramid of Giza code the value of Pi?
2. Finding the state of rest of a hanging mass.
3. Why do some planets appear to move backwards in the night sky?
4. Drawing interesting curves with a spirograph.
5. Using the Leontieff model to predict prices in a closed economy and devising a strategy for Monopoly?
6. Distinguishing knots by their colour polynomials.
7. What is the nature of cubic curves.
8. Tomography and solving the Challenger puzzle
9. Using voting polynomials to judge the fairness of the constitution.
11. Modelling how falling snow packs.
12. Doing arithmetic with Egyptian fractions.
13. Set Theory: cardinals and ordinals, basic paradoxes and some ZF
14. Theory of Games (like NIM), Grundy-Sprague, Conway
15. Theory and Practice of Cubic Splines
17. Hilbert's Metric and positive contraction mappings in a Banach Space
18. Theory of real symmetric matrices
19. Finite Ramsey theory in combinatorics and graph theory
20. Symmetry groups of the Platonic solids
21. Applications of quaternions to describing rotations
23. Classical convergence tests for infinite series of numbers and functions
24. Regular plane tesselations and their symmetries
25. Pade approximation and applications to ODE - Bulirsch-Stoer method
26. Pseudo-primes and Carmichael Numbers. Also RSA
27. Genetic Algorithms and their applications
28. Classical geometry of the Euclidean, Elliptic and Hyperbolic planes
29. A survey of methods for primality checking
30. Convergence theory for Fourier series. Fejer, Gibbs' Phenomenon
31. Classification of regular and semi-regular solids
32. Coset enumeration, Todd-Coxeter
33. Non-negative matrices. Perron-Frobenius
34. Maximum entropy methods in image enhancement
35. Archimedes' work on mensuration and the method of Exhaustion
36. Linkages, in particular linkages for drawing any algebraic curve
37. Map projections and their properties
38. Bell's theorem and the EPR paradox
39. Construction of Fundamental regions for the Modular group
40. Colouring maps on surfaces of positive genus
41. Geometric constructability of the regular polygons
42. Magic Squares
43. The spectrum of a graph. Relation to regular and line graphs
44. Canonical transformations. Hamilton-Jacobi theory. Noether's theorem
45. General Theory of Orthogonal Polynomials
46. The Banach-Tarski paradox
47. Linear ODE with periodic coefficients. Floquet theory. Applications
48. Spirals in Nature, or anything from d'Arcy Thompson "On Growth and Form"
49. The analysis of the running time of the Euclidean Algorithm (see Knuth)
50. Period-doubling and Chaos. Feigenbaum results. The Verhulst process
51. Solitons and the KdV equation. Lax representation and the KdV hierarchy
52. Mathematical methods for computer Tomography
53. Thermographic strategy and Games. Berlekamp et al. : Winning Ways
54. Survey of results on the location of roots of polynomials
55. The Toom-Cook Algorithm and fast multiplication
56. The revival of number theory in the seventeenth century. Fermat
58. The Fatou and Julia sets. From Blanchard's article in Bull AMS 1984
59. Spinors in quantum mechanics. Representation theory of $O(3)$ and $O(1,3)$
60. Digraphs and Tournaments. Harary etc "Structural Models", Wiley 1965
61. Steiner systems. Construction of Steiner systems with the Mathieu group
62. Theory of the symmetrical heavy top under gravity
63. Numerical solutions of Burger's equation as a turbulence model
64. The physics of neutron Stars. Shapiro & Teukolsky "Black Holes,..."
66. Numerical integration of stiff ODE. Particularly BDF methods
68. The geometry of numbers, with application to Diophantine approximation
69. Mathematical cryptography
70. Mobius transformations
71. Some (or all) proofs of PNT
72. Calculus of Variations and Control Theory
73. Rolling, slipping and sliding. 3-d motion of two bodies in contact
74. Mechanics in sport
75. Vibration of circular cylinders
76. The cosmic microwave background (how to dry Shrodinger's cat)
77. Monte Carlo techniques for the quantum many-body problem
78. Calculations of the binding energies of atomic nuclei
79. Action-angle variables in Hamiltonian and quantum mechanics
80. Soliton solutions of the Sine-Gordon equation
81. Differential geometry and curved space
82. Quantum mechanics of angular momentum
83. Probability models for population growth processes
84. Random walks and their applications
85. Latin squares in experimental design and group theory
86. Bus timetabling
87. Limits and/or continuity in topological spaces
88. Development of mathematical reasoning in Primary schools
89. Disappearance of mathematical reasoning in Universities
90. Fermat’s Last Theorem (19th century work, and updates?)
91. Paradoxes of the infinite
92. 2-dimensional airfoil theory
93. Irrationality and Transcendentality of pi and e
94. Methods of numerical integration
95. Intuitionism
96. Theory of shock waves
97. Accident proneness
98. Perfect numbers
99. Traffic flow
100. Godel’s theorem
101. Frequency stability criterion for dynamical systems
102. Skolem’s Paradox
103. Boole and his algebra
104. The dissection of rectangles into squares
105. Axiom of choice and some of its implications
106. Computation of the characters of the symmetric groups
107. Convergence and stability of multistep methods for ODE
108. Development of probability theory from C16 to the present day
109. Similarity methods for differential equations
110. Fixed point theorems
111. Fibonacci numbers
112. Fuzzy set theory and logic
113. Bessel’s equation and Bessel functions
114. The Siefert-Van Kampen theorem in topology
115. The sound of stringed instruments
116. Hall subgroups and finite soluble groups
117. A numerical study of problems in population dynamics
118. Numerical modelling of the performance of road traffic noise barriers
119. Modelling the performance of diffraction gratings
120. Value-added measures in education
121. Studies of the oscillating water column wave energy device
122. Development of teaching software for the finite element method
123. Periodically forced spherical pendulum
124. Coexistence in competition models in ecology
125. The mathematical modelling of disease propagation
126. Some aspects of the dynamics of the Duffing oscillator and its attractors
127. Iteration under real and complex cubic maps
128. Modelling a car exhaust system
129. Preprocessing techniques in Linear and Integer Programming
130. Vehicle routing and scheduling
131. The Leech lattice and coding
132. Heuristic search methods in Permutation groups of small degree
133. Limited storage Quasi-Newtonian algorithms for unconstrained optimisation
134. The Computer Straight Forecasting Formula used in horse racing
135. Constrained interpolation
136. Data Compression
137. Triangulation of Point Sets
138. Re-merging Feigenbaum trees in dynamical systems
139. Finding all the vertices of a convex polyhedron $Ax \leq b$, $x \geq 0$
140. Dynamic thermal modelling of buildings
141. Fractal image compression
142. Investigation into graduates early careers at Shell Int. Petroleum
143. Graphical display of polyhedra - a library
144. Handicapping systems
145. Stellar Structure
146. Canonical forms for Matrices
147. Alternating multilinear algebra
148. Circulant Matrices
149. The group of upper triangular matrices
150. Continued, Egyptian and Farey fractions
151. Rational Approximation
152. deBruijn sequences and arrays
153. Error-correcting codes
154. Isonemal fabrics
155. Asymptotic expansions
156. Construction of the Reals
157. Homotopy and covering spaces
158. Regulated Functions
159. Stability theory for ODE
160. Fundamental regions for the Modular group
161. Geometry of Gaussian and linear optics
162. Airy's map projection. Balance of error
163. Cartan's geometrical mechanics
164. The Gyroscope
165. Recurrence and van der Waerden's theorem
166. The sun, the moon and Stonehenge
167. Convexity and Matrix Games
168. Tournament designs
169. Residue arithmetic and Gauss-Jordan algorithm
170. Singular value decomposition
171. Calculating knot invariants by Maple
172. Contour integrals in Maple
173. Computer classification of conic sections and quadric surfaces
174. The pendulum - with the pivot oscillating vertically
175. Job scheduling
176. Flow in a network

More?
MATHEMATICS RESEARCH PROJECT IDEAS
SUITABLE FOR HIGH SCHOOL AND COLLEGE STUDENTS

This list is a copy of the list Possible Science Fair Mathematics Projects which is maintained by Afton H. Cayford, at The University of British Columbia.

What follows is a selection of ideas for science projects. In most cases only a very brief outline is presented (sometimes with a reference) in order to leave students plenty of scope for what they do. It is not expected that the problem stated will necessarily be the project. These are ideas intended to get people thinking (they are in no particular order).

1. At certain times charities call households offering to pick-up used items for sale in their stores. They often do a particular geographical area at a time. Their problem, once they know where the pick-ups are, is to decide on the most efficient routes to make the collection. Find out how they do this and investigate improving their procedure. A similar question can be asked about snow plows clearing city streets, or garbage collection. References: Euclidean tours, Chinese postman problem - information can be found in most books on graph theory but one of particular interest is "Introduction to Graph Theory" by G. Chartrand.

2. How should one locate ambulance stations, so as to best serve the needs of the community? The reference given above may help.

3. An International Food Group consists of twenty couples who meet four times a year for a meal. On each occasion, four couples meet at each of five houses. The members of the group get along very well together; nonetheless, there is always a bit of discontent during the year when some couples meet more than once! Is it possible to plan four evenings such that no two couples meet more than once? There are many problems like this. They are called combinatorial designs. Investigate others.

4. How does the NBA work out the basketball schedule? How would you do such a schedule bearing in mind distances between locations of games, home team advantage etc.? Could you devise a good schedule for one of your local competitions?

5. How do major hospitals schedule the use of operating theatres? Are they doing it the best way possible so that the maximum number of operations are done each day?

6. Investigate "big" numbers. What is a big number? The following examples might guide your investigation. A bank is robbed of 1 million loonies. How long it would take to move that many? How much it would weigh? How much space would it take up? How big a swimming pool do you need to contain all the blood in the world? Is $10^{100}$ very big? What is the biggest number anyone has ever written down (check the Guiness book of world records over the last few years)? How did this number come about?
7. Build a physical model based on dissections to prove the Pythagorean Theorem. Build an exhibit on the Pythagorean theorem but with "The semicircle on the hypotenuse ..."

8. What is the fewest number of colours needed to colour any map if the rule is that no two countries with a common border can have the same colour. Who discovered this? Why is the proof interesting? What if Mars is also divided into areas so that these areas are owned by different countries on earth. They too are coloured by the same rule but the areas there must be coloured by the colour of the country they belong to. How many colours are now needed? Reference: Joan Hutchinson, ...

9. Study the golden mean, its appearance in art, architecture, biology, and geometry, and its connection with continued fractions, fibonacci numbers. What else can you find out? What is the Golden Mean?

10. Study the regular solids (platonic and Archimedean), their properties, geometry, and occurrence in nature (e.g. virus shapes, fullerene molecules, crystals). Build models.

11. Study the cycloid curve: its tautochrone and brachistochrone properties and its history. Build models.

12. Infinity comes in different "sizes". What does this mean? How can it be explained? References: Refer to either of the Dover paperbacks, "Theory of Sets" by Kamke, and "The Continuum and other types of Serial Order" by Huntington, or any book on Set Theory.

13. Investigate visual representations of different finite numbers. For example, if p is a prime with 100 digits, then if 1 and p are on the same line segment, with p say 6 inches to the right of 1, then p^{1/2}, the square root of p, is about 10^{-50} inches to the right of 1, less than one atom away. (And it's by inspecting the lattice points in the p^{1/2} x p^{1/2} array that one proves that p is the sum of two squares!) Investigate further.


15. Study games and winning strategies - maybe explore a game where the winning strategy is not known. Analyze subtraction games (nim-like games in which the two players alternately take a number of beans from a heap, the numbers being restricted to a given subtraction set). References: E.R. Berlekamp, J.H. Conway, R.K. Guy, "Winning Ways", Academic Press, London (this book contains hundreds of othr games for which the complete analysis is unknown eg. Toads and Frogs); R. Guy (editor), "Combinatorial Games", Proceedings of
16. Most computers these days can handle sound one way or another. They store the sound as a sequence of numbers. Lots of numbers. 40,000 per second, say. What happens when you play around with those numbers? eg. Add 10 to each number. Multiply each number by 10. Divide by 10. Take absolute values. Take one sound, and add it to another sound (i.e. add up corresponding pairs of numbers in the sequences). Multiply them. Divide them. Take one sound, and add it to shifted copies of itself. Shuffle the numbers in the sequence. Turn them around backwards. Throw out every third number. Take the sine of the numbers. Square them. For each mathematical operation, you can play the resulting sound on the computers speakers, and hear what change has occurred. A little bit of programming, and you can get some very bizarre effects. Then try to make sense of this from some sort of theory of signal processing.


18. Investigate the space shuttle’s failed attempt to put a tethered satellite into orbit.

19. Draw, and list any interesting properties of, various curves: evolutes, involutes, roulette, pedir curves, conchoids, cissoids, strophoids, caustics, spirals, ovals, ... Reference: Cundy and Rollett, “Mathematical Models”, Oxford (which has lots of other ideas, too); E.H. Lockwood, “A Book of Curves”, Cambridge; and there’s also a book by Yates, “Curve Tracing”?

20. Make a family of polyhedra, e.g., the Archimedean solids, or Deltahedra (whose faces are all equilateral triangles), or equilateral zonohedra, or, for the very ambitious, the 59 Isocahedra. Reference: See any Coxeter revision of Rouse Ball’s “Mathematical Recreations and Essays” (which is full of many ideas). There’s also Coxeter, DuVal, Flather and Petrie, “The 59 Isocahedra”, U of Toronto Press; Magnus J Wenninger, “Polyhedron Models”, Cambridge, 1971; and Doris Schattschneider and Wallace Walker, “M.C. Escher Kaleidocycles”, Pomegranate Art Books, 1987.

21. Find as many triangles as you can with integer sides and a simple linear relation between the angles. What about the special case when the triangle is right-angled?

22. Find out all you can about the Fibonacci Numbers, 0, 1, 1, 2, 3, 5, 8, ...

23. Find out all you can about the Catalan Numbers, 1, 1, 2, 5, 14, 42, ...

24. What is Morley’s triangle? Draw a picture of the 18 Morley triangles associated with a given triangle ABC. Find the 18 more for each of the triangles BHC, CHA, AHB, where H is the orthocentre of ABC. Discover the relation with the 9-point circle and deltoid (envelope of the Simson or Wallace line).


27. Ten frogs sit on a log - 5 green frogs on one side and 5 brown frogs on the other with an empty seat separating them. They decide to switch places. The only moves permitted are to jump over one frog of a different colour into an empty space or to jump into an adjacent space. What is the minimum number of moves? What if there were 100 frogs on each side? Coming up with the answers reveals interesting patterns depending on whether you focus on colour of frog, type of move, or empty space. Proving it works is interesting also - it can lead to recursion, there is also a simple proof that is not immediately obvious when you start. Look for and explore other questions like this - one of the most famous is the Tower of Hanoi.


29. There is a well-known device for illustrating the binomial distribution. Marbles are dropped through the top and encounter a number of pins before dropping into cells where they are distributed according to the binomial distribution. By changing the position of the pins one should be able to get other kinds of distributions (bimodal, skewed, approximately rectangular, etc.) Explore.

30. Build rigid and non-rigid geometric structures. Explore them. Where are rigid structures used? Find unusual applications. This could include an illustration of the fact that the midpoints of the sides of a quadrilateral form a parallelogram (even when the quadrilateral is not planar). Are there similar things in three dimensions?

31. Build a true scale model of the solar system - but be careful because it cannot be contained within the confines of an exhibit. Illustrate how you would locate it in your town. Maybe even do so!!

32. Build models to illustrate asymptotic results such as Stirling's formula or the prime number theorem.

33. What is/are Napier's bones and what can you do with it/them?

34. Covering a chessboard with dominoes so that no two dominoes overlap and no square on the chessboard is uncovered. Consider (a) a full chessboard, (b) a chessboard with one square removed (impossible - why?), (c) a chessboard with two adjacent corners removed, (d) one with two opposite corners removed (possible or impossible?), (e) A chessboard with any two squares removed. What
about using shapes other than dominoes (e.g., 3 one-by-one squares joined together)? What about chessboards of different dimensions? Reference: "Polyominoes" by Solomon W. Golomb, pub. Charles Scribner's Sons.

35. Build models showing that parallelograms with the same base and height have the same areas (is there a 3-dimensional analogue?). This can lead to a purely visual proof of the Pythagorean theorem. The formula for the area of a circle can also be presented in this way. Reference: H. R. Jacobs, "Mathematics a Human Endeavor", 3rd ed, p 38.

36. Use Monte Carlo methods to find areas (rather than using random numbers, throw a bunch of small objects onto the required area and count the numbers of objects inside the area as a fraction of the total in the rectangular frame) or to estimate pi.

37. Find pictures which show that \(1 + 2 + \ldots + n = (1/2)n(n+1)\), that \(1^2 + 2^2 + \ldots + n^2 = (1/6)n(n+1)(2n+1)\), and that \(1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2\). How many other ways can you find to prove these identities? Is any one of them "best"?

38. What is fractal dimension? Investigate it by examining examples showing what happens when you double the scale to (a) lines (b) areas (c) solids (d) the Koch curve.

39. Knots. What happens when you put a knot in a strip of paper and flatten it carefully? When is what appears to be a knot really a knot? Look at methods for drawing knots.

40. Is there an algorithm for getting out of 2-dimensional mazes? What about 3-dimensional? Look at the history of mazes (some are extraordinary). How would you go about finding someone who is lost in a maze (2 or 3-dimensional) and wandering randomly? How many people would you need to find them?

41. Investigate the history of pi and the many ways in which it can be approximated. Calculate new digits of pi - see Peter Borwein's homepage to discover what this means.

42. What is game theory all about and where is it applied?

43. Construct a Kaleidoscope. Investigate its history and the mathematics of symmetry.

44. Consider tiling the plane using shapes of the same size. What's possible and what isn't. In particular it can be shown that any 4-sided shape can tile the plane. What about 5 sides? Look for books and articles by Grunbaum and Shepherd, and check the Martin Gardner books.

45. Explore Penrose tiles and discover why they are of interest.
46. Investigate the Steiner problem - one application of which is concerned with the location of telephone exchanges to minimize costs.

47. Look for new strategies for solving the travelling salesman problem.

48. Explore Egyptian fractions.

49. How do computer bar codes (the ones you see on everything you buy) work? This is an example of coding theory at work. Investigate coding theory - there are many books with titles like "an introduction to coding theory" (this is not about secret codes). References: Joe Gallian, "How computers can read and correct ID numbers", Math Horizons, Winter, 1993, p14-15; Joe Gallian, "Assigning Driver's License Numbers", Mathematics Magazine, 64 (1991), 13-22; and Joe Gallian, "Math on Money", Math Horizons, November, 1995, p10-11.

50. The Art Gallery problem: What is the least number of guards required to watch over all paintings in an art gallery? The guards are positioned at specific locations and collectively must have a direct line of sight to every point on the walls. Reference: Alan Tucker, "The Art Gallery Problem", Math Horizons, Spring, 1994, p24-26.

51. The Parabolic Reflector Microphone is used at sporting events when you want to be able to hear one person in a noisy area. Investigate this; explaining the mathematics behind what is happening.

52. There is a traditional Chinese way of illustrating the Pythagorean theorem using paper. Investigate and make models.

53. Use PID (proportional-integral-differential) controllers and oscilloscopes to demonstrate the integration and differentiation of different functions.

54. Try the "Monty Hall" effect. Behind one of three doors there is a prize. You pick door #1, he shows you that the prize wasn't behind door #2 and then gives you the choice of switching to door #3 or staying with #1, what should you do? Why should you switch? Make an exhibit and run trials to "show" this is so. Find the mathematical reason for the switch.

55. Look at the ways different bases are used in our culture and how they have been used in other cultures. Collect examples: time, date etc. Demonstrate how to add using the Mayan base 20, maybe compare to trying to add with Roman numerals (is it even possible?)

56. Explore the history and use of the Abacus.


58. Explore magic tricks based in Mathematics (again see the article about Persi Diaconis).
59. Investigate compass and straight-edge constructions - showing what's possible and discussing what's not. For example, given a line segment of length one can you use them straight edge and compass to "construct" all the radicals?

60. Chaos and the double pendulum.

61. There are several books that have a variety of articles that can be used to generate projects:
   * Paul Hoffman, "Archimedes Revenge", Ballantine Books
   * Nancy Casey and Mike Fellows (1993). "This is mega-mathematics: stories and activities for mathematical thinking, problem-solving and communication", The Los Alamos National Laboratory, Los Alamos, New Mexico
   * Arthur L Loeb, "Concepts and Images, Visual Mathematics"
   * And then of course there are all the Martin Gardner books.

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More?

1. I Cut, You Choose

One method that children use to fairly divide a piece of cake or candy is to have one person do the cutting and the other do the choosing. This encourages the person doing the cutting to cut the pieces as evenly as possible so that the person doing the choosing does not get a larger piece. How would this method work for three people? Four? Five? Can you develop a pattern or algorithm that would provide a method for any number of people?

2. Latin Squares

It's a perpetual wonder that mathematical theories developed with no useful purpose in mind except to satisfy a mathematician curiosity, often and most unexpectedly apply not only to other parts of mathematics but to other sciences and real world problems. Non-euclidean geometries became an integral part of the General Theory of Relativity. Group Theory and the theory of Semigroups of Operators serve as an important tool in Quantum Mechanics. Encryption
algorithms that underly security of internet transactions are based on finding huge prime numbers.

Orthogonal latin squares have been considered by Euler probably for their entertaining value. He posed the problem of 36 officers: Is it possible to arrange 36 officers, each having one of six different ranks and belonging to one of six different regiments, in a square formation 6 by 6, so that each row and each file shall contain just one officer of each rank and just one from each regiment?

Besides having an exciting history, latin squares developed into a very respectable branch of mathematics with various applications. Starting with the early twentieth century, latin square found statistical applications as experimental designs (BIB - balanced incomplete block - designs.) The theory of designs depends on such abstract mathematical tools as finite fields and finite geometries.

Latin squares are good for scheduling round-robin tournaments. As a matching procedure, latin squares relate to problems in Graph Theory, job assignment (or Marriage Problem), and, more recently, processor scheduling for massively parallel computer systems. Algorithms for solving the Marriage Problem are also used in Linear Algebra to reduce matrices to block diagonal form.

References

2. W.W. Rouse Ball and H.S.M. Coxeter, Mathematical Recreations and Essays, Dover, 1987
4. Encyclopædia Britannica

On Internet

1. Partying With A Latin Square (modern applications: parallel computations, conflict resolution)
2. Latin Squares and Related Topics (Collection of references with quotes, cryptographic application)
3. The "Magic Carpet" Approach to Understanding Magic Squares
4. Graeco-Latin Squares

3. Liavati – A famous mathematical work in the form of poems – written by Bhaskaracharya – in 1150
The title is the name of the daughter of the mathematician.